

Assignment Quiz 2
October 10, 2001

Instructor: B.L. Daku
Time: 15 minutes
Aids: None

Name: _____
Student Number: _____

1. Determine the output of the LTI system defined by

$$h[n] = 2^n u[-n-2],$$

if the input is given by

$$x[n] = 2u[n-2] - 3u[n-9].$$

$$y[n] = \underbrace{\sum_{k=-\infty}^{\infty} 2u[n-2] 2^{n-k} u[k-n-2]}_{y_1} - \underbrace{\sum_{k=-\infty}^{\infty} 3u[n-9] 2^{n-k} u[k-n-2]}_{y_2}$$

$$\begin{aligned} y_1: & n \leq 0 \\ y_1 &= \sum_{k=2}^{\infty} 2 \cdot 2^{n-k} \\ &= 2^{n+1} \left(\frac{(\frac{1}{2})^2 - (\frac{1}{2})^{\infty}}{1 - \frac{1}{2}} \right) \\ &= 2^{n+1} \left(\frac{1}{2} \right) = 2^n \end{aligned}$$

$$\begin{aligned} y_2: & n \geq 0 \\ y_2 &= \sum_{k=9}^{\infty} 3 \cdot 2^{n-k} \\ &= 3 \cdot 2^n \left(\frac{(\frac{1}{2})^9 - (\frac{1}{2})^{\infty}}{1 - \frac{1}{2}} \right) \\ &= 3 \cdot 2^n \left(\frac{1}{2} \right)^8 = 3 \cdot 2^{n-8} \end{aligned}$$

$2^n = 1$ when $n = 0$

$$y_2: n \leq 7$$

$$\begin{aligned} y_2 &= \sum_{k=9}^{\infty} 3 \cdot 2^{n-k} \\ &= 3 \cdot 2^n \left(\frac{(\frac{1}{2})^9 - (\frac{1}{2})^{\infty}}{1 - \frac{1}{2}} \right) \\ &= 3 \cdot 2^n \left(\frac{1}{2} \right)^8 = 3 \cdot 2^{n-8} \end{aligned}$$

$$3 \cdot 2^{n-8} = \frac{3}{2}$$

$2^{n-8} = 1$ $n = 7$ critical point

$$y = y_1 - y_2$$

$$y[n] = \begin{cases} 2^n - 3 \cdot 2^{n-8} & n \leq 0 \\ 1 - 3 \cdot 2^{n-8} & n \leq 7 \\ -1/2 & n \geq 7 \end{cases}$$



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UNIVERSITY OF SASKATCHEWAN
ELECTRICAL ENGINEERING 455.3

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1. Analytically determine the following discrete-time convolution.

$$y[n] = \alpha^n u[n] * \beta^n u[n-2], \quad |\alpha| < 1, |\beta| < 1$$

$$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k] \beta^{n-k} u[n-k-2]$$

$$n < 2$$

$$n-k > 0 \\ n > k$$

$$n \geq 2$$

$$n-k > 0 \\ k > 2$$

$$n < 2$$

$$y[n] = 0$$

$$\begin{aligned} y[n] &= \sum_{k=2}^n \alpha^k \beta^{n-k} \\ &= \alpha^2 \beta^{n-2} \sum_{k=2}^n \left(\frac{\alpha}{\beta} \right)^{k-2} \\ &= \alpha^2 \beta^{n-2} \left(\frac{1 - (\frac{\alpha}{\beta})^{n-1}}{1 - \frac{\alpha}{\beta}} \right) \\ &= \alpha^2 \beta^{n-2} \frac{1 - (\frac{\alpha}{\beta})^{n-1}}{1 - \frac{\alpha}{\beta}} \end{aligned}$$

$$\begin{aligned} y[n] &= \sum_{k=2}^n \alpha^k \beta^{n-k} \\ &= \alpha^2 \beta^{n-2} \sum_{k=2}^n \left(\frac{\alpha}{\beta} \right)^{k-2} \\ &= \alpha^2 \beta^{n-2} \frac{(\frac{\alpha}{\beta})^{n-1} - (\frac{\alpha}{\beta})^0}{1 - \frac{\alpha}{\beta}} \\ &= \alpha^2 \beta^{n-2} \frac{(\frac{\alpha}{\beta})^{n-1} - 1}{1 - \frac{\alpha}{\beta}} \end{aligned}$$

$$y[n] = \frac{\alpha^{n-2} \beta^2 - \alpha^2 \beta^{n-1}}{1 - (\alpha/\beta)} = \frac{\alpha^2 \left(\frac{\beta}{\alpha} \right)^2 - \left(\frac{\beta}{\alpha} \right)^{n+1}}{1 - \frac{\beta}{\alpha}}$$

